Self-stabilizing Connected Components

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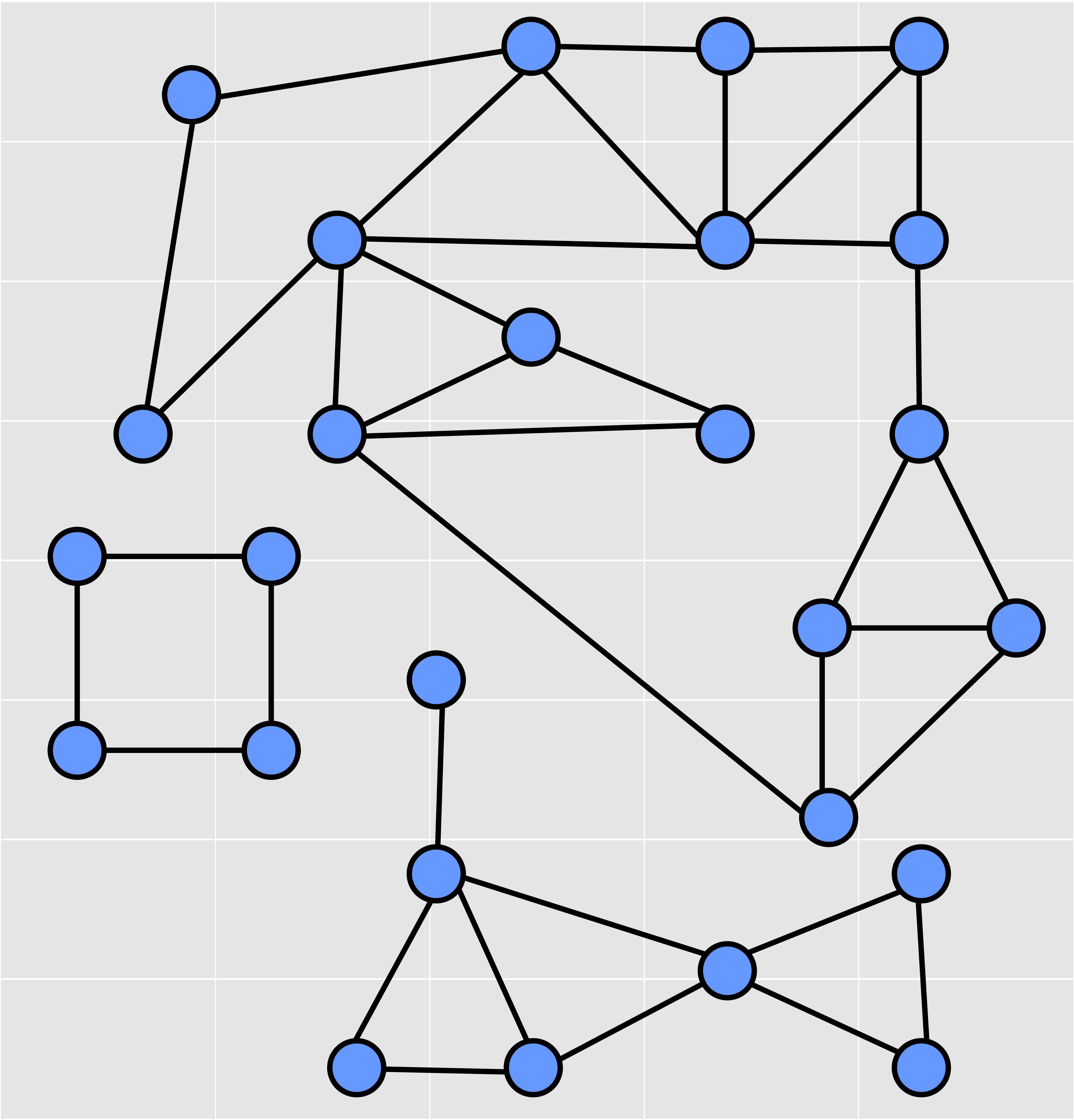
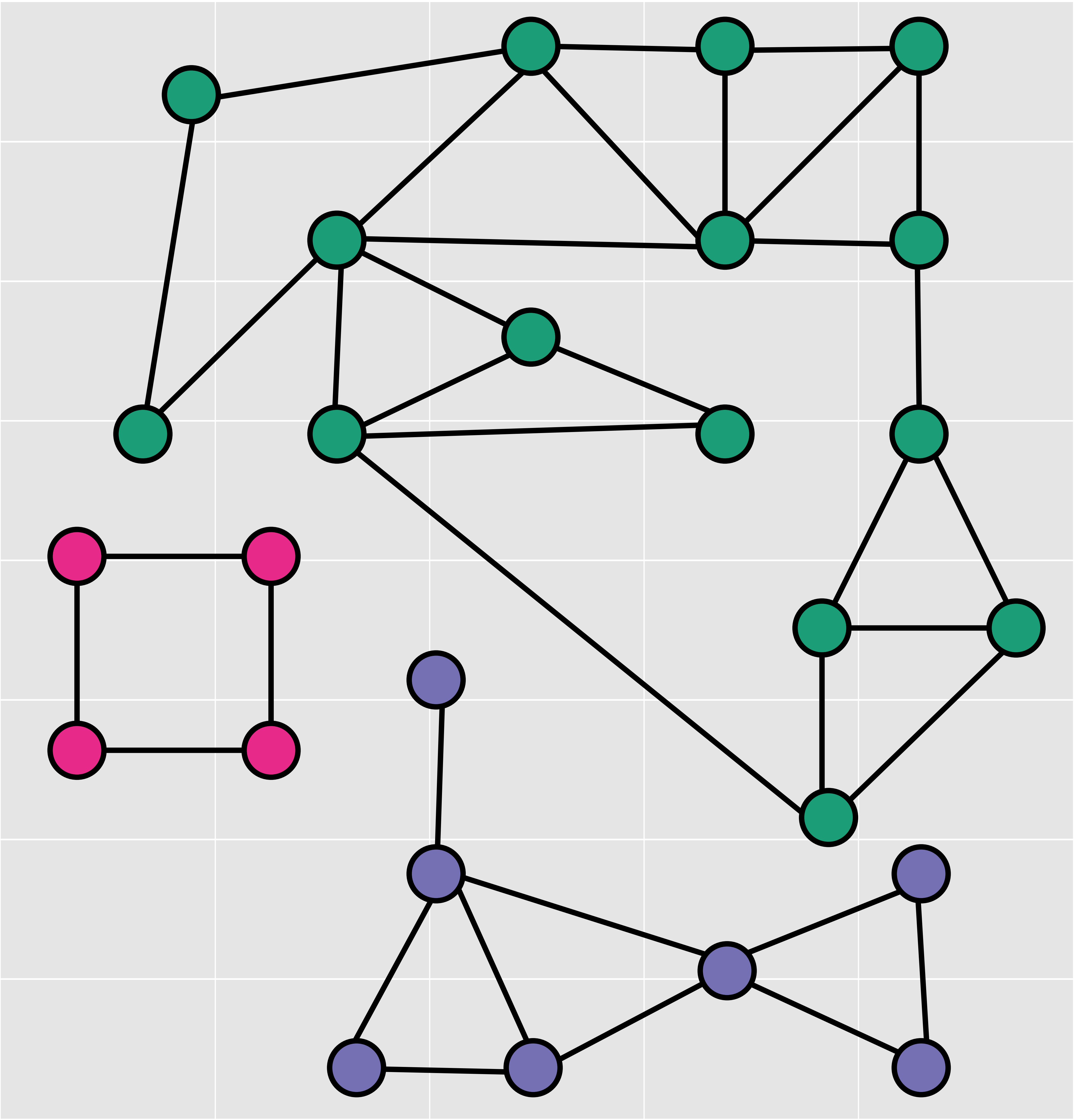
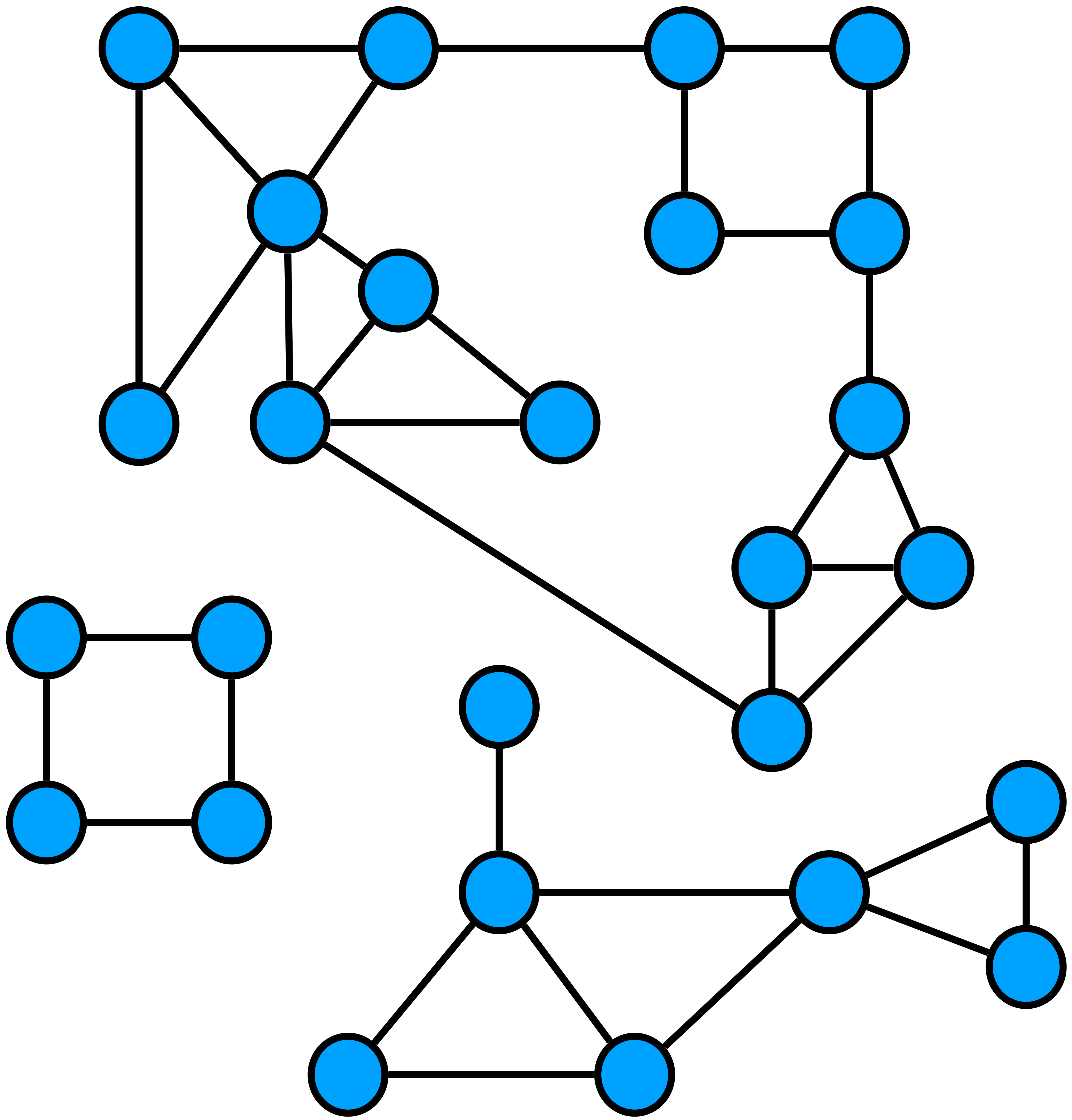
Graph Connected-Components





Label Propagation (LP) Algorithm



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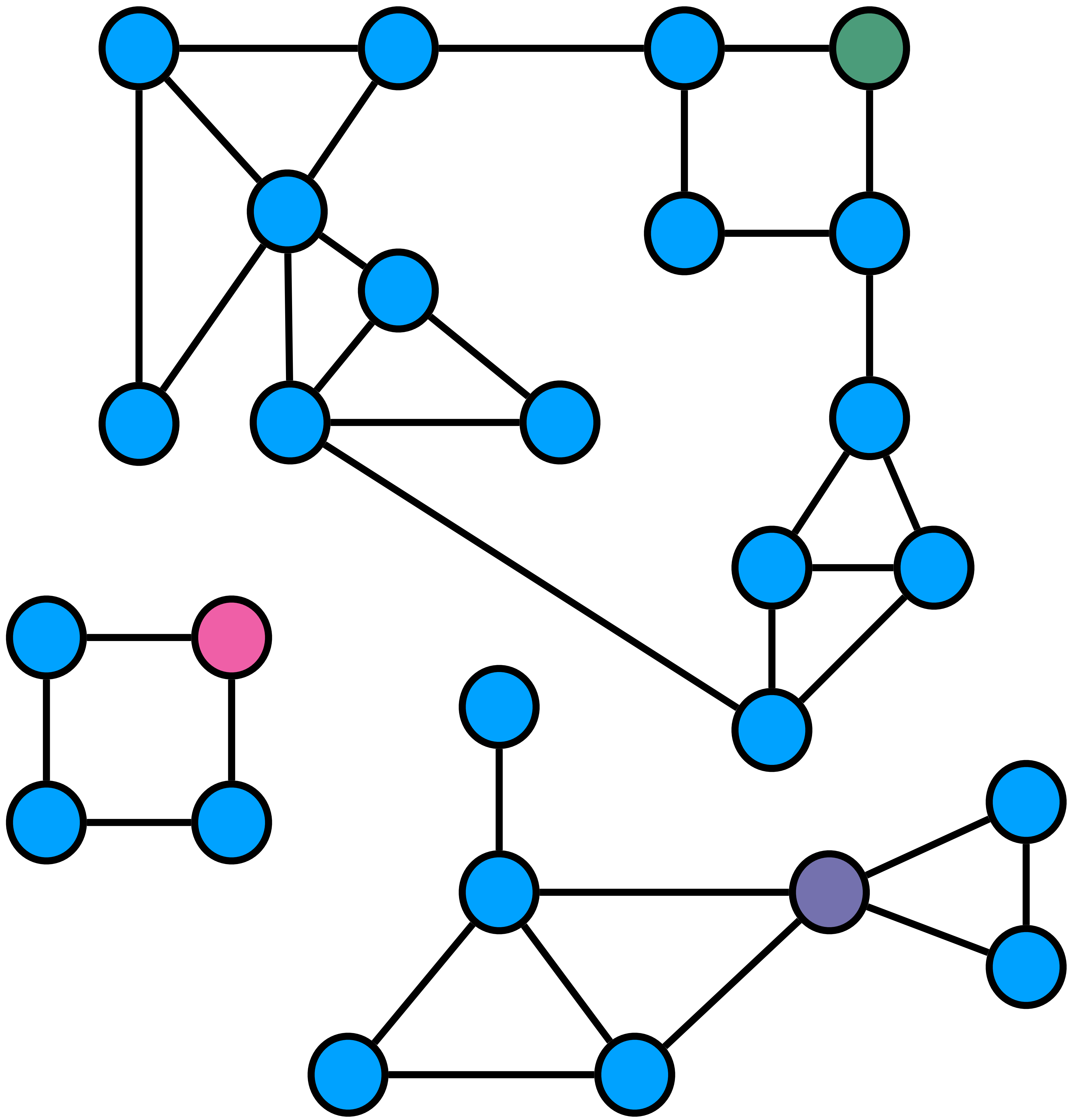

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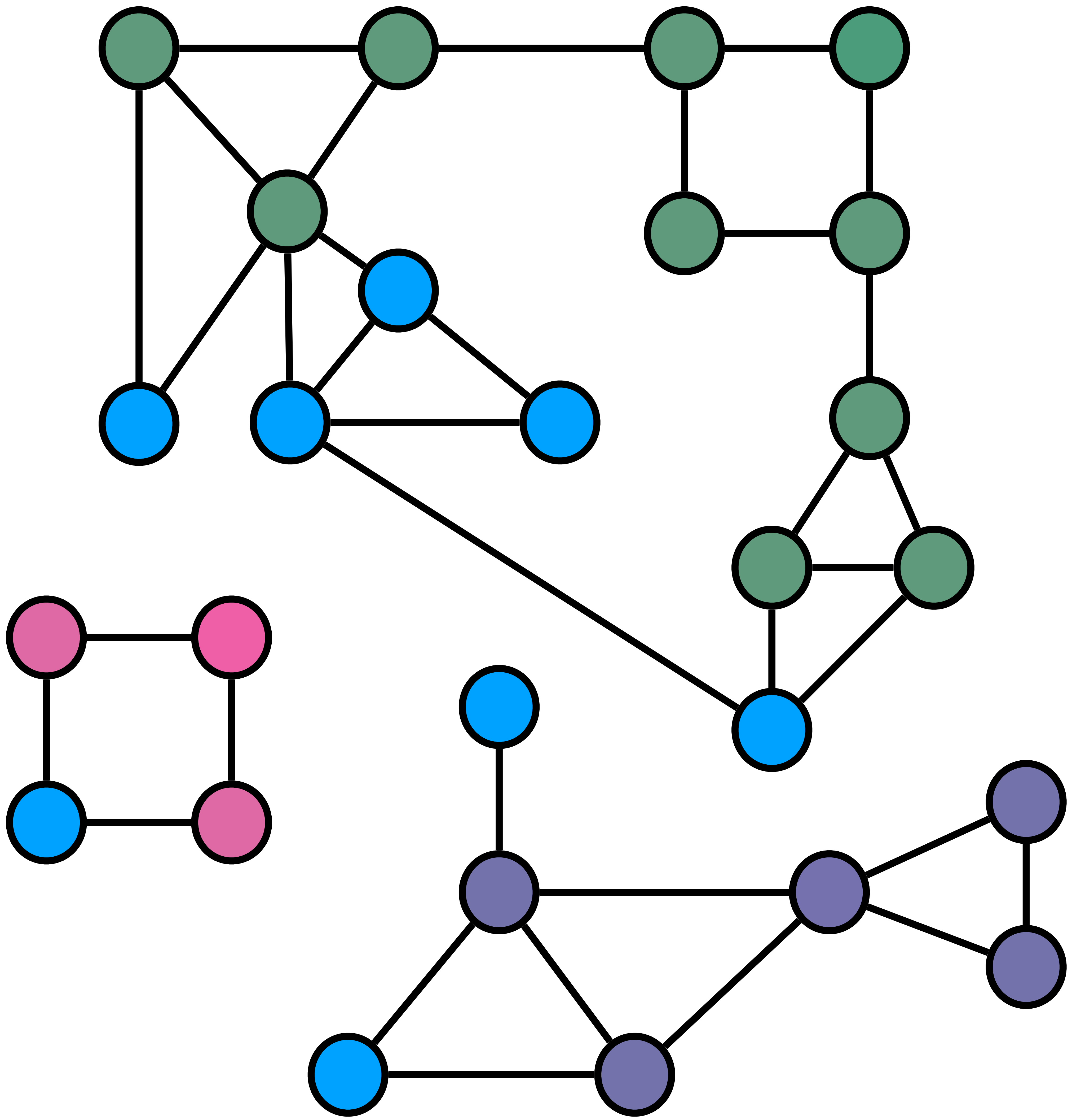

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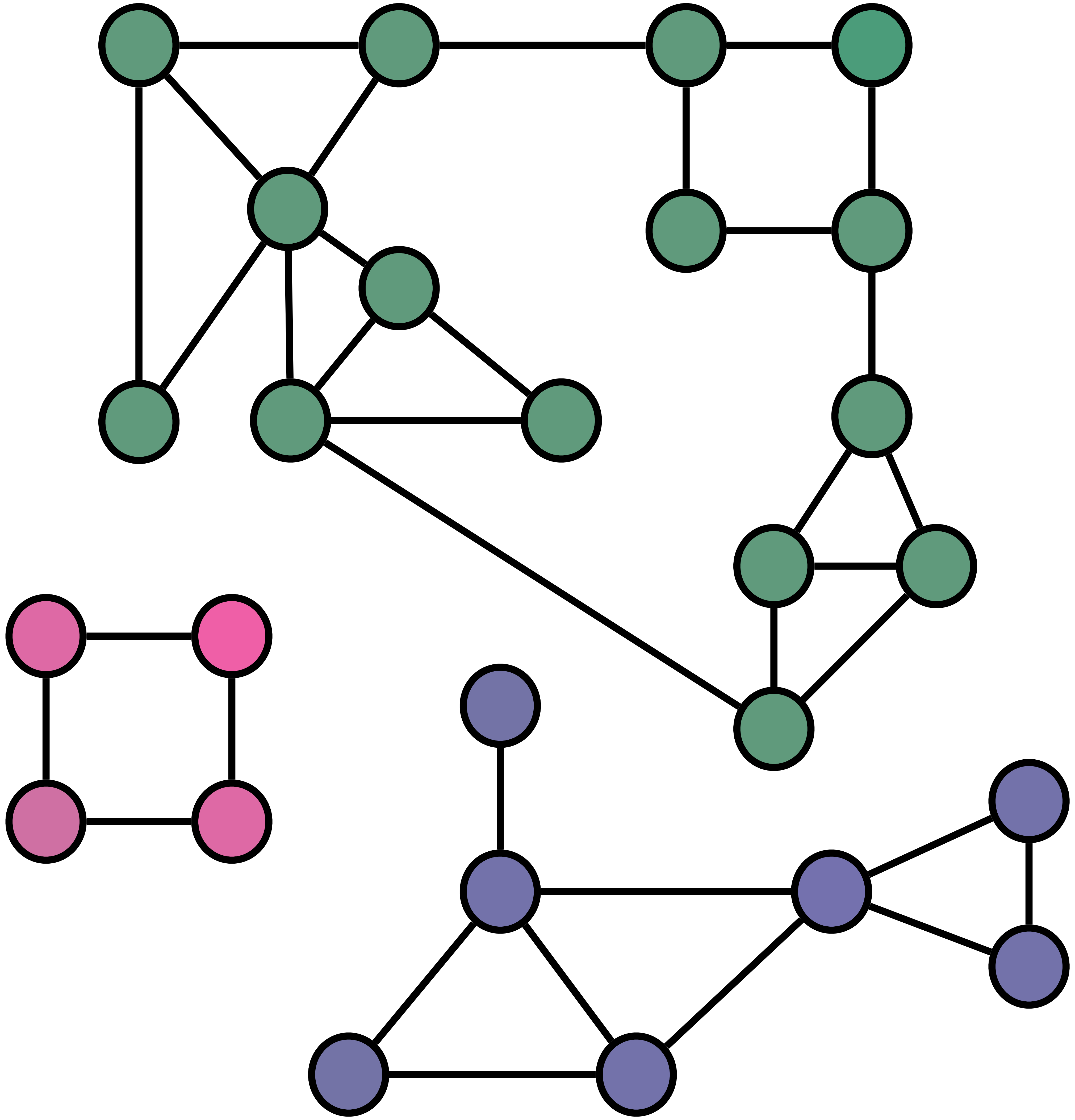
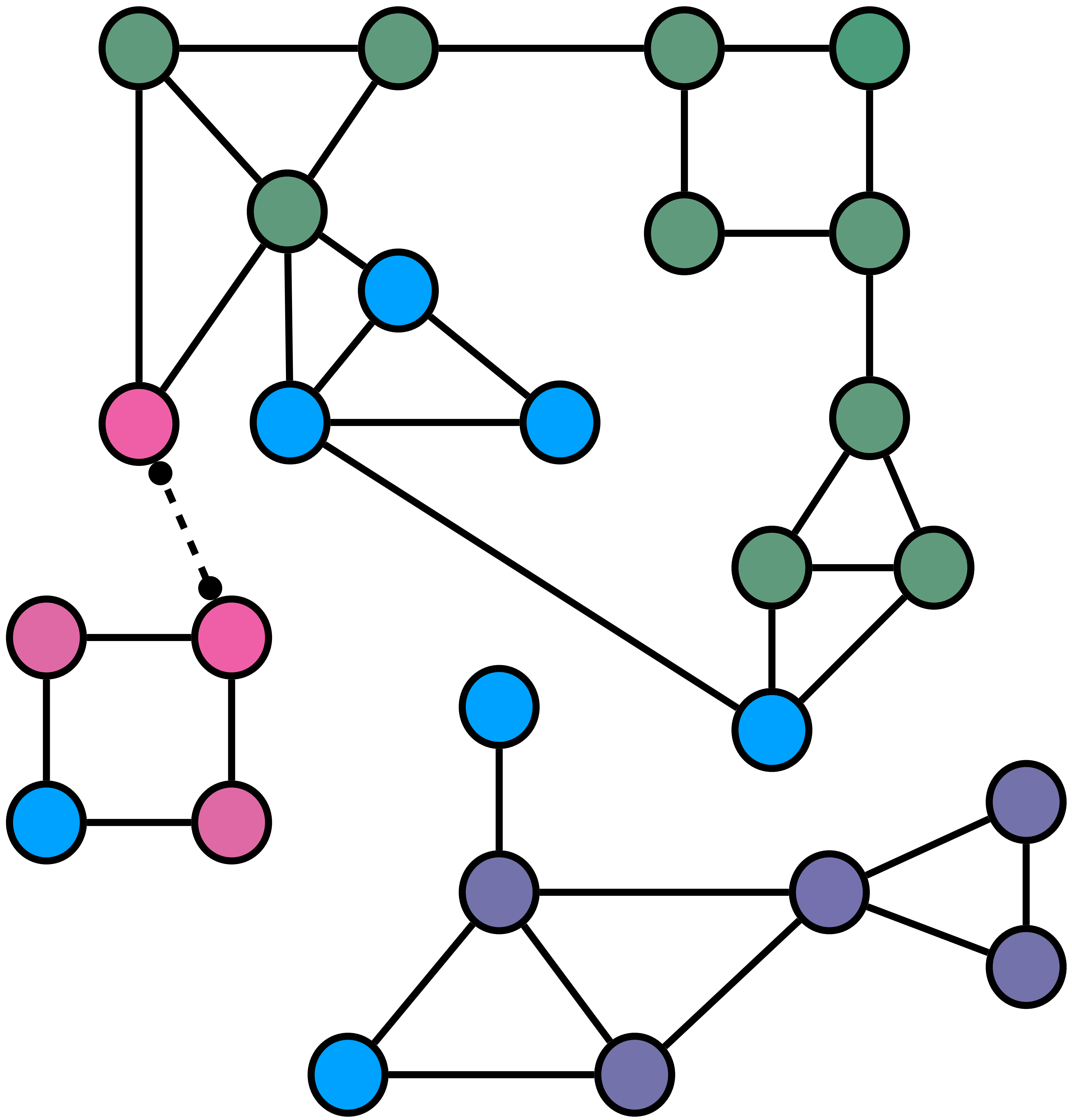
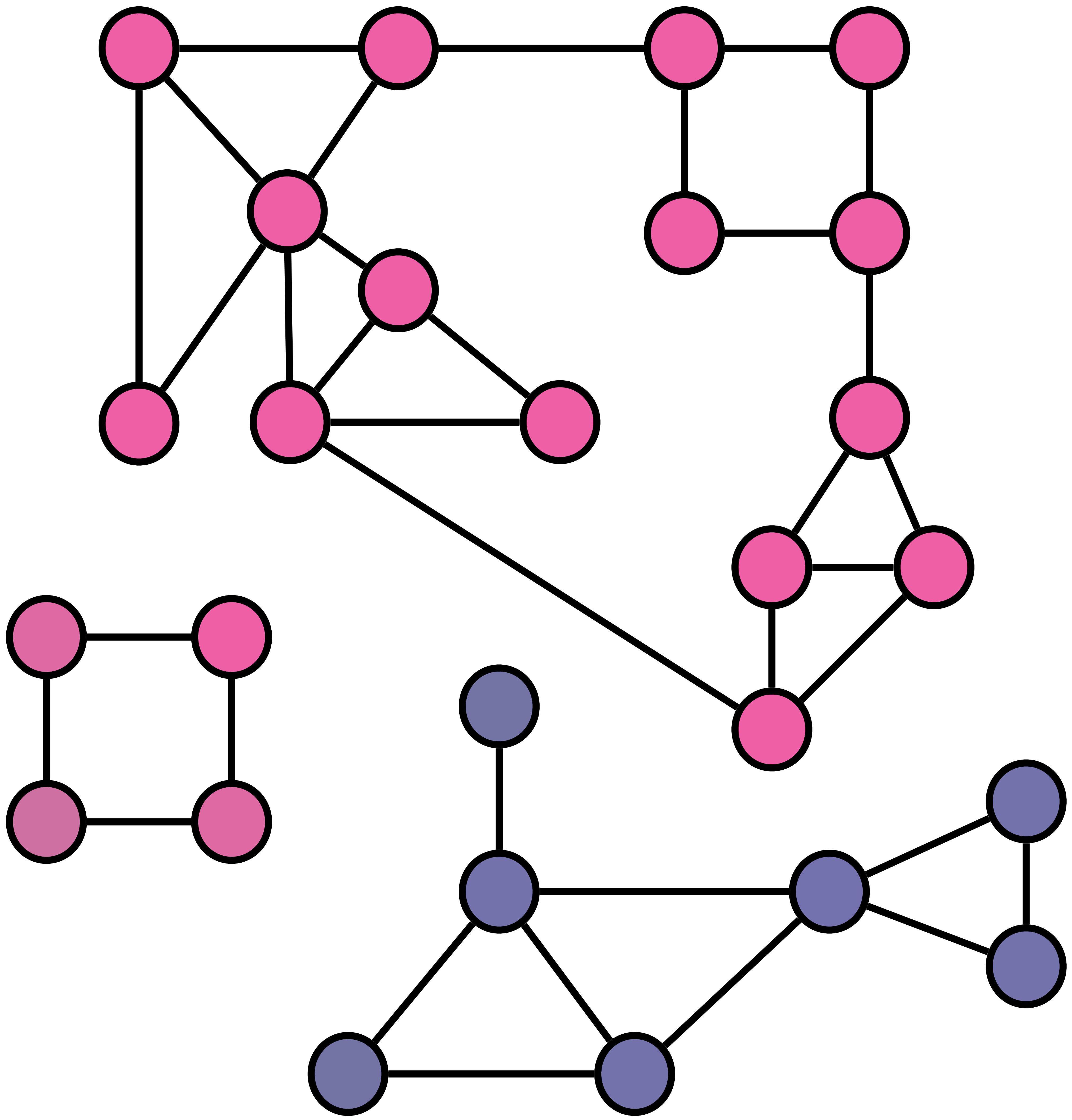


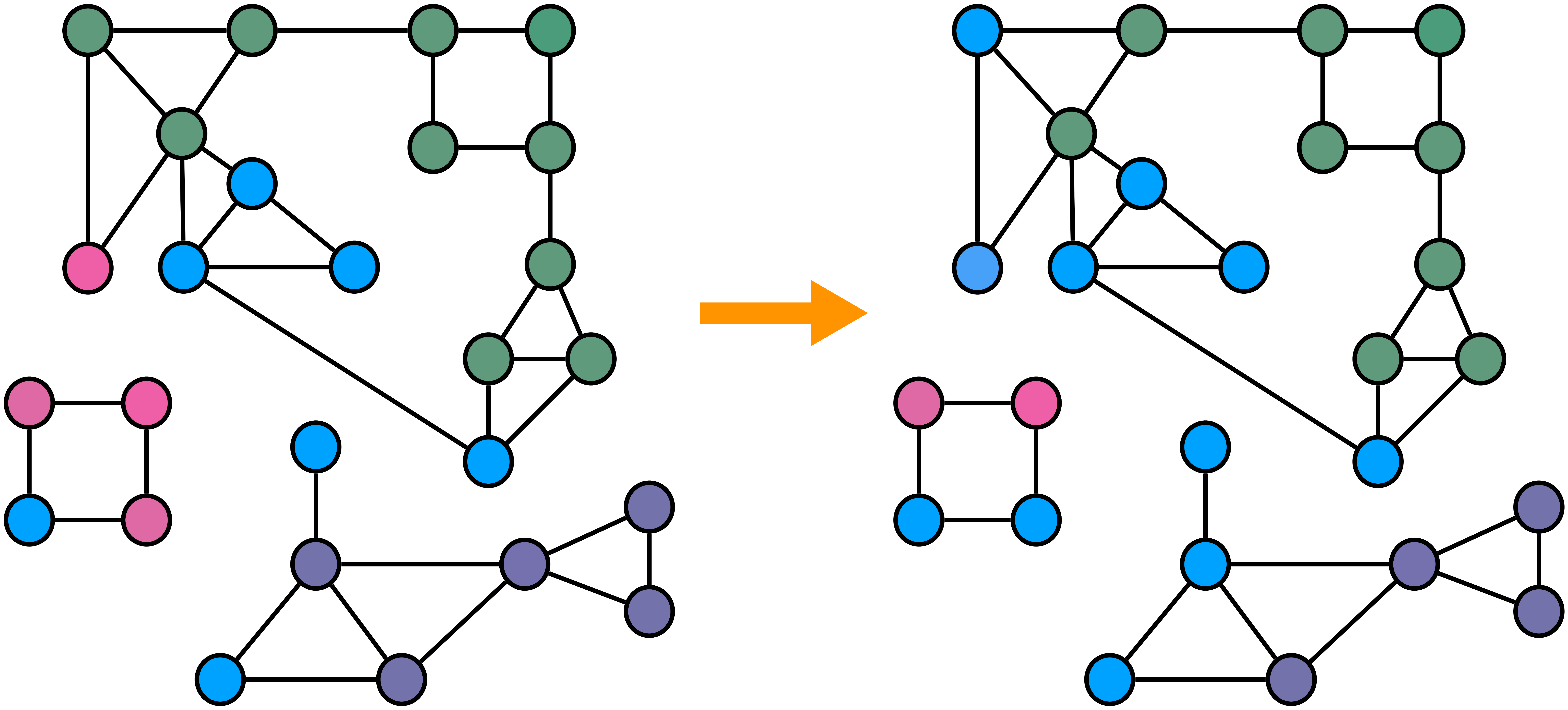
Impact of Faults in LP Algorithm



Impact of Faults in LP Algorithm



Self-stabilizing Connected-Components



Arbitrary State (valid or invalid)

Guaranteed Valid State

0. Label-Propagation for Graph Connected-components Problem

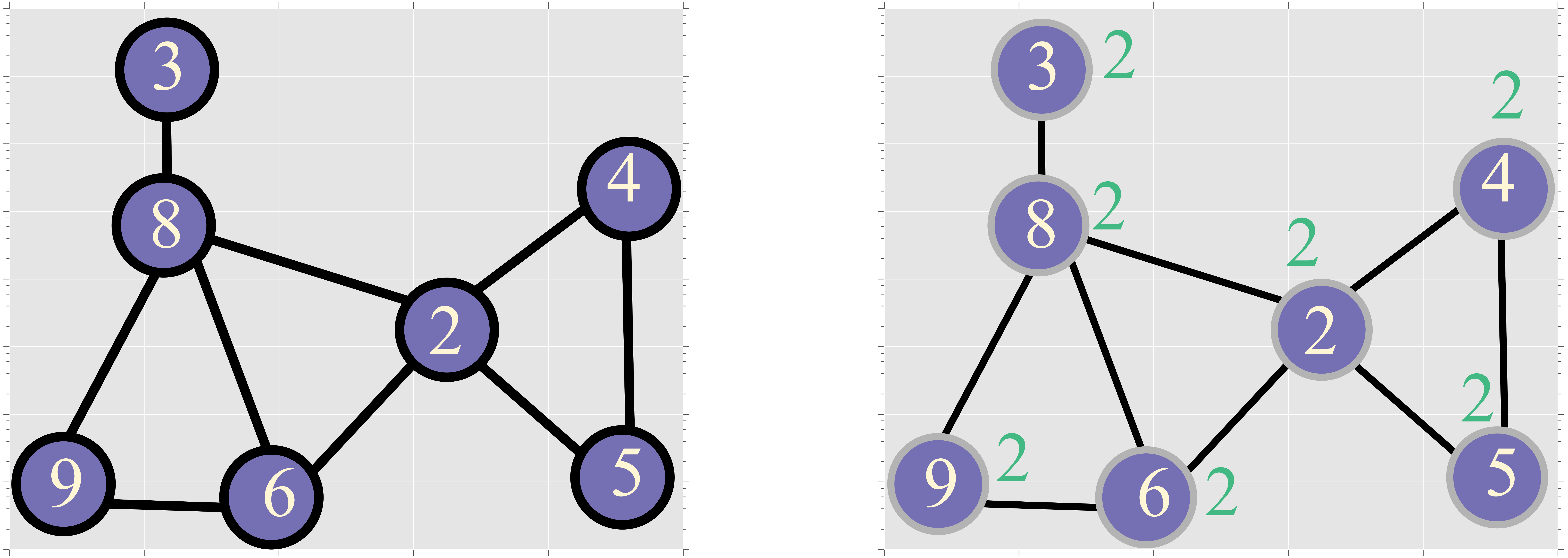
1. Self-correcting Connected Components

— Sao, Green, Jain, Vuduc (FTXS'16)

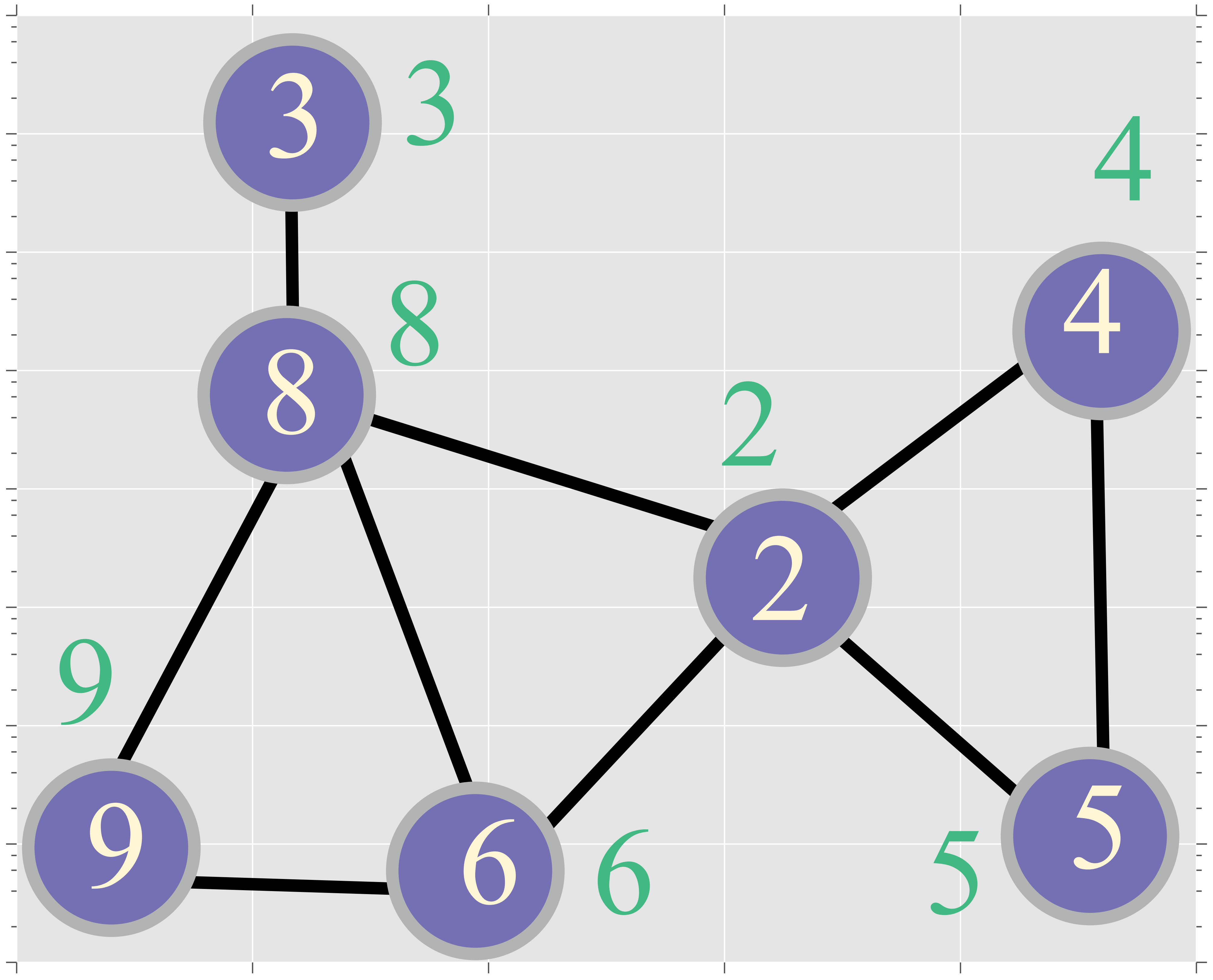
2. Self-stabilizing Connected Components

— Sao, Engalmann, Eswar, Green, Vuduc (FTXS'19)

Label Propagation (LP) Algorithm-0

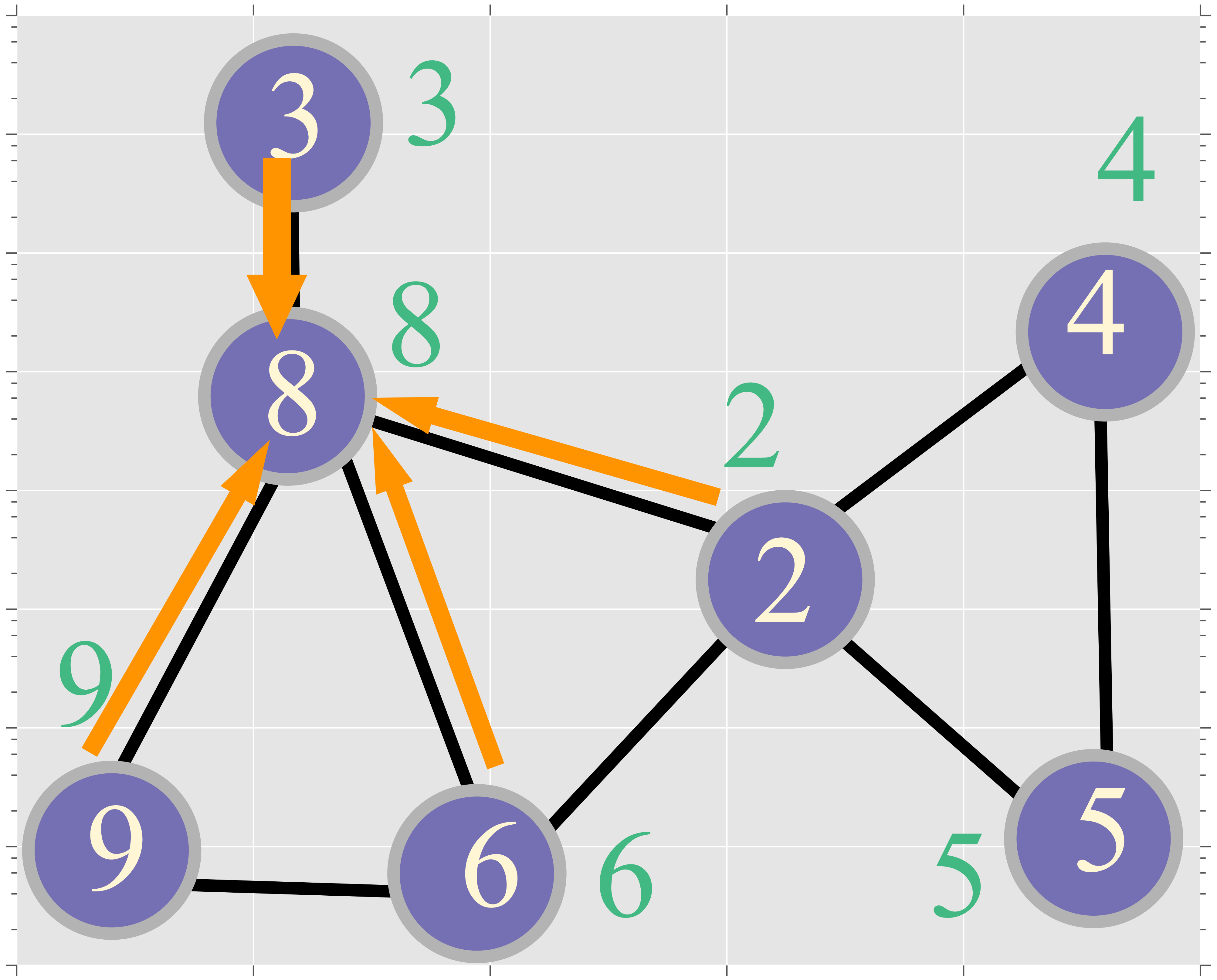


LP-Initialization



$$L^0(v) \leftarrow v$$

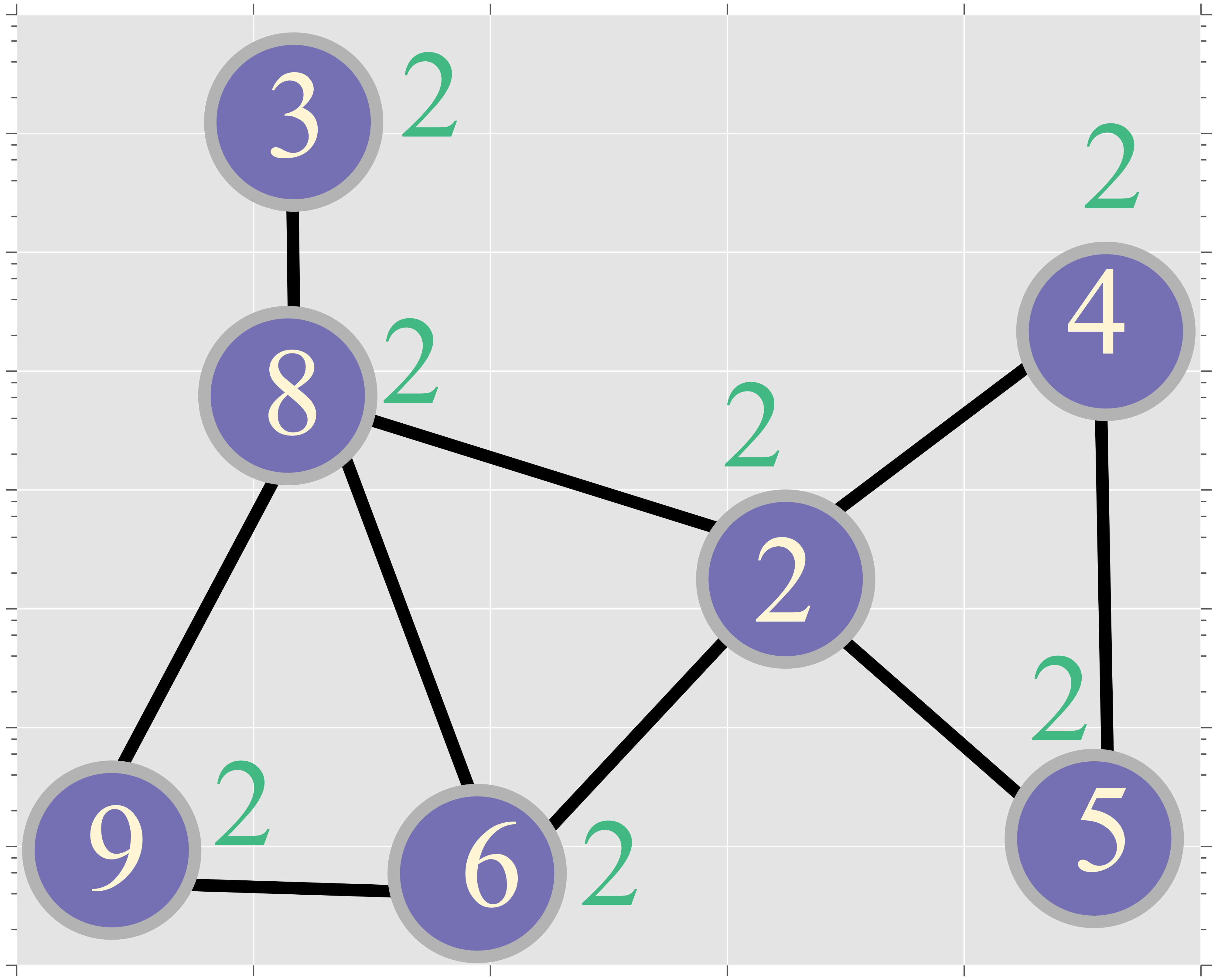
LP-Update



$$L^0(v) \leftarrow v$$

$$L^{i+1}(v) \leftarrow \min_{u \in \mathcal{N}(u)} L^i(u)$$

LP-Termination



$$L^0(v) \leftarrow v$$

$$L^{i+1}(v) \leftarrow \min_{u \in \mathcal{N}(u)} L^i(u)$$

$$\textit{Final Label}: \quad L^{\infty}(v)$$

Terminates when there are no more label Changes

0. Label-Propagation for Graph Connected-components Problem

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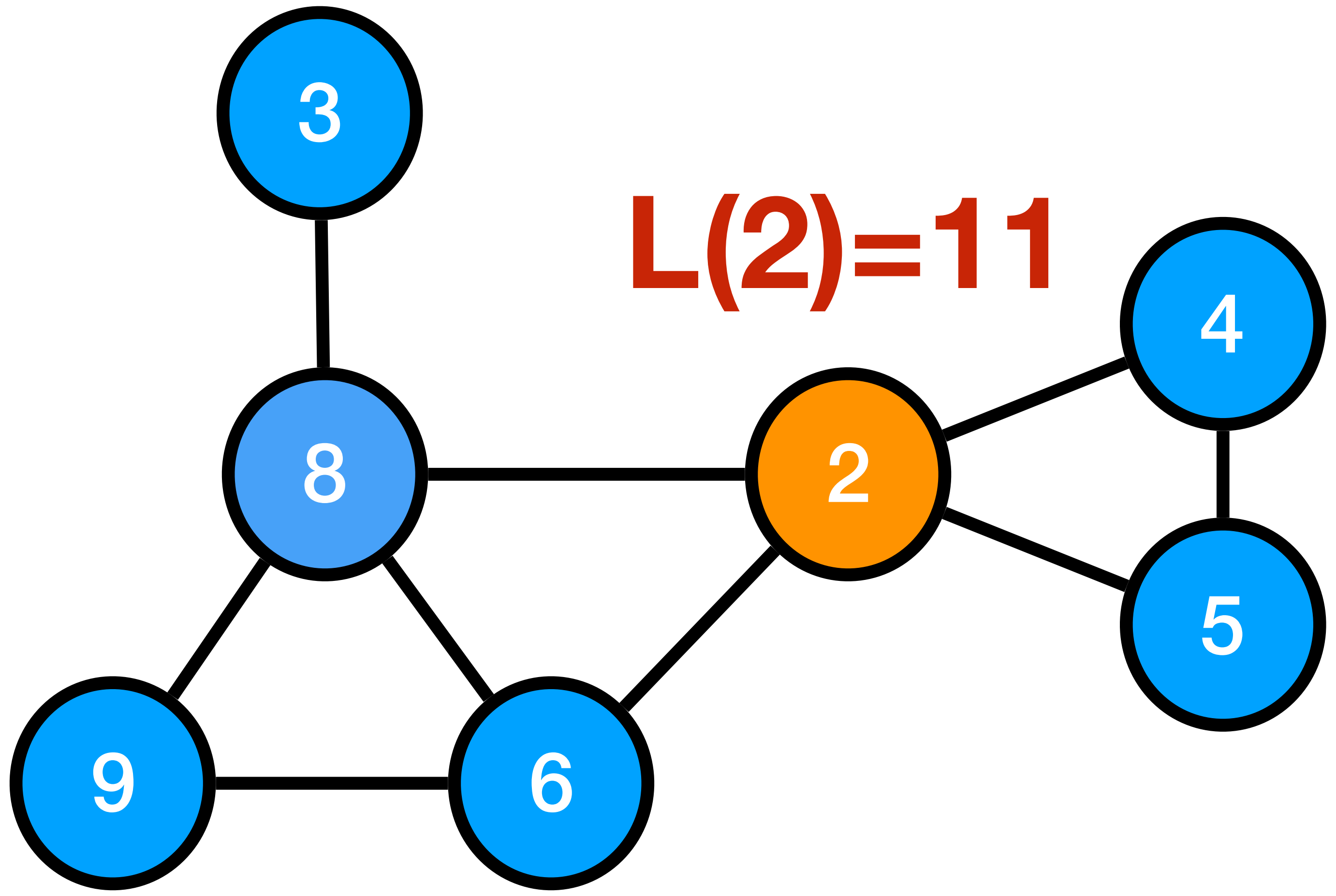
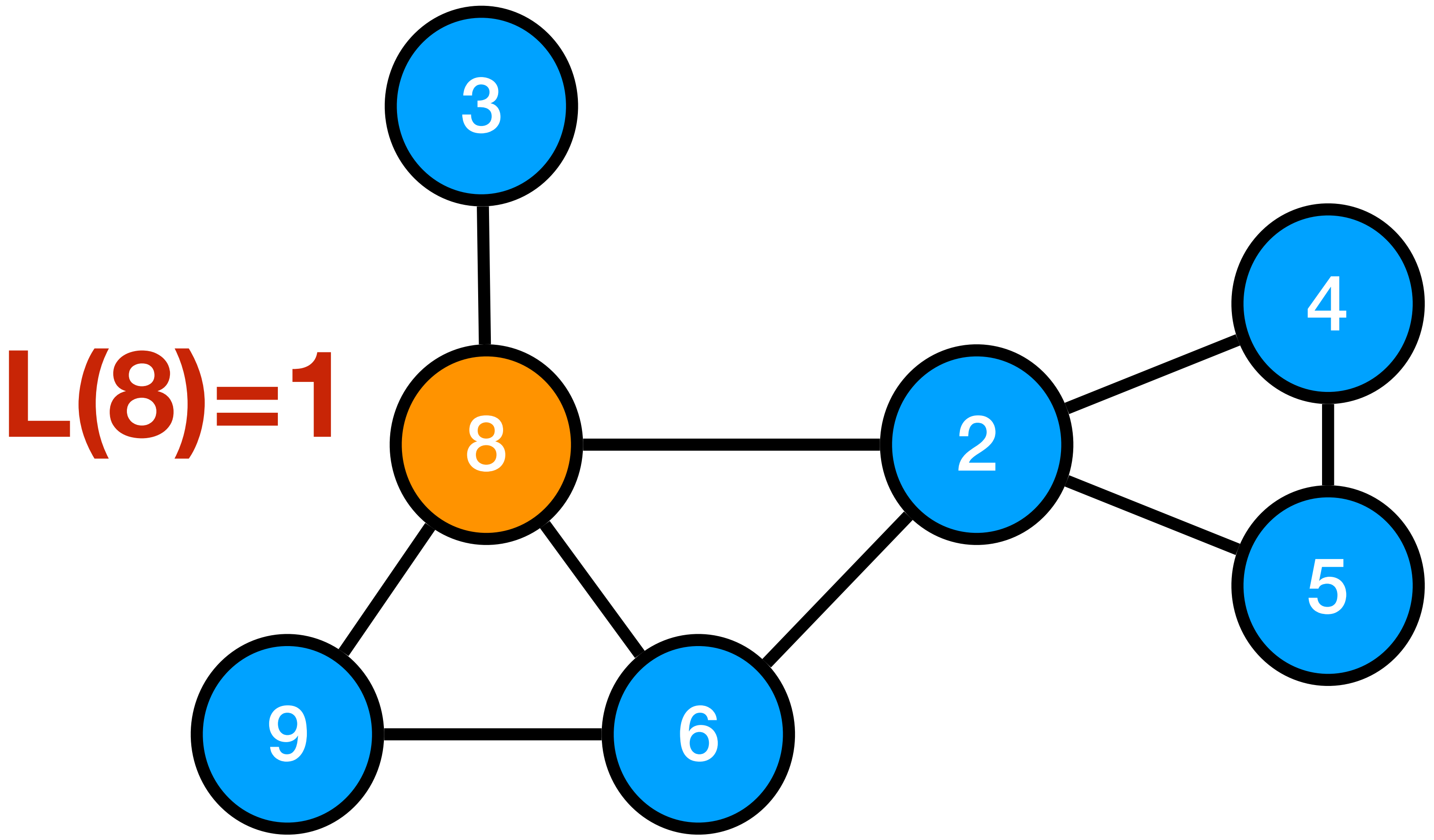
Valid States



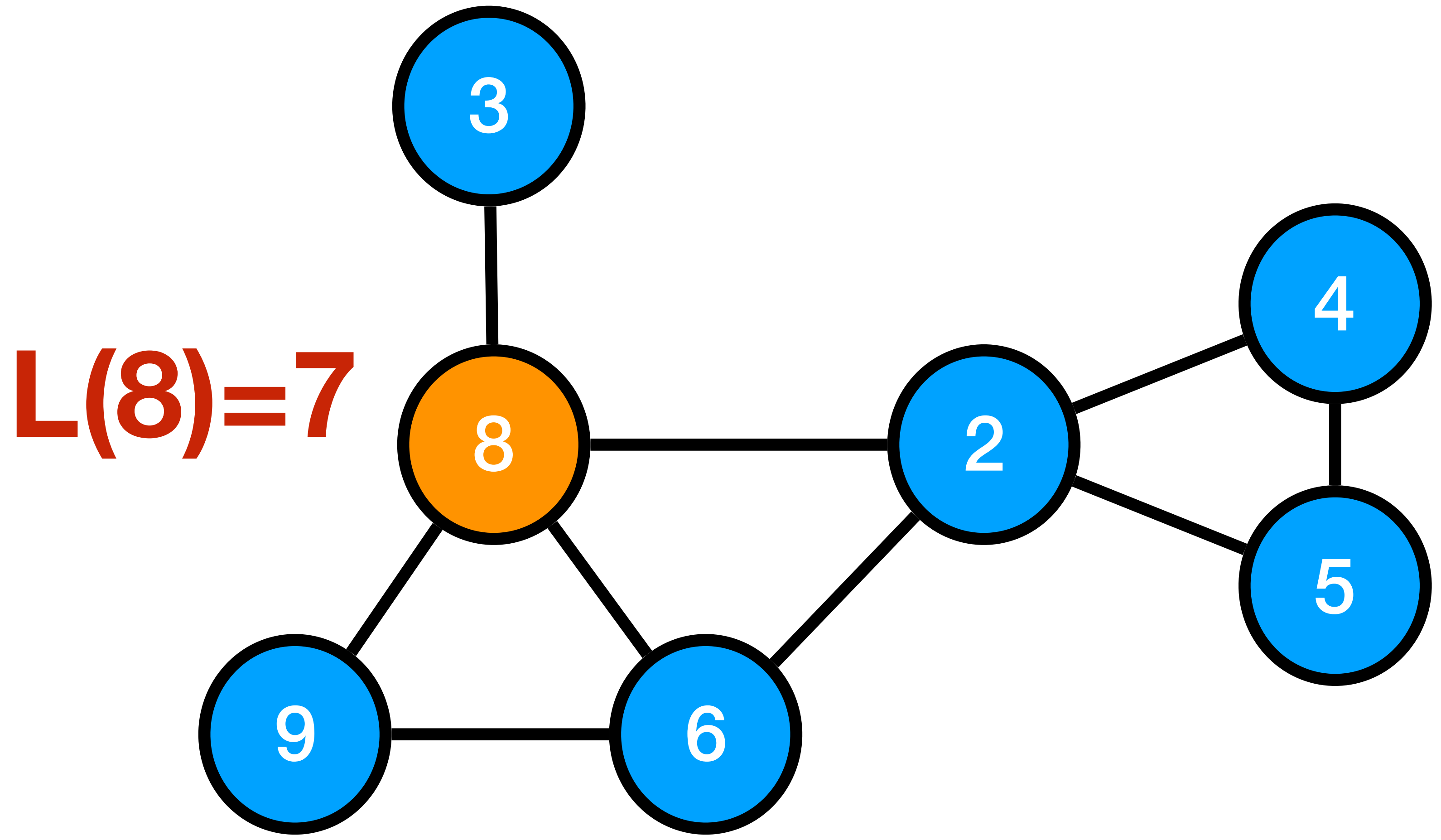
$$L^{\infty}(v) \leq \boldsymbol{L(v)} \leq v \qquad \forall \boldsymbol{v} \in \boldsymbol{V}$$

Valid States



$$L^{\infty}(v) \leq \boldsymbol{L(v)} \leq v \qquad \forall \boldsymbol{v} \in \boldsymbol{V}$$

Valid States



$$L^{\infty}(v) \leq L(v) \leq v \qquad \forall v \in V$$

Unknown

Self-correcting Connected Components

$$P(v)$$ ***v*** acquired its current label from ***P(v)***

$$L^{i+1}(v) \leftarrow \min_{u \in \mathcal{N}(u)} L^{i}(u)$$

Verifying this requires **O(V+E)** operations

0. Label-Propagation for Graph Connected-components Problem

1. Self-correcting Connected-Components

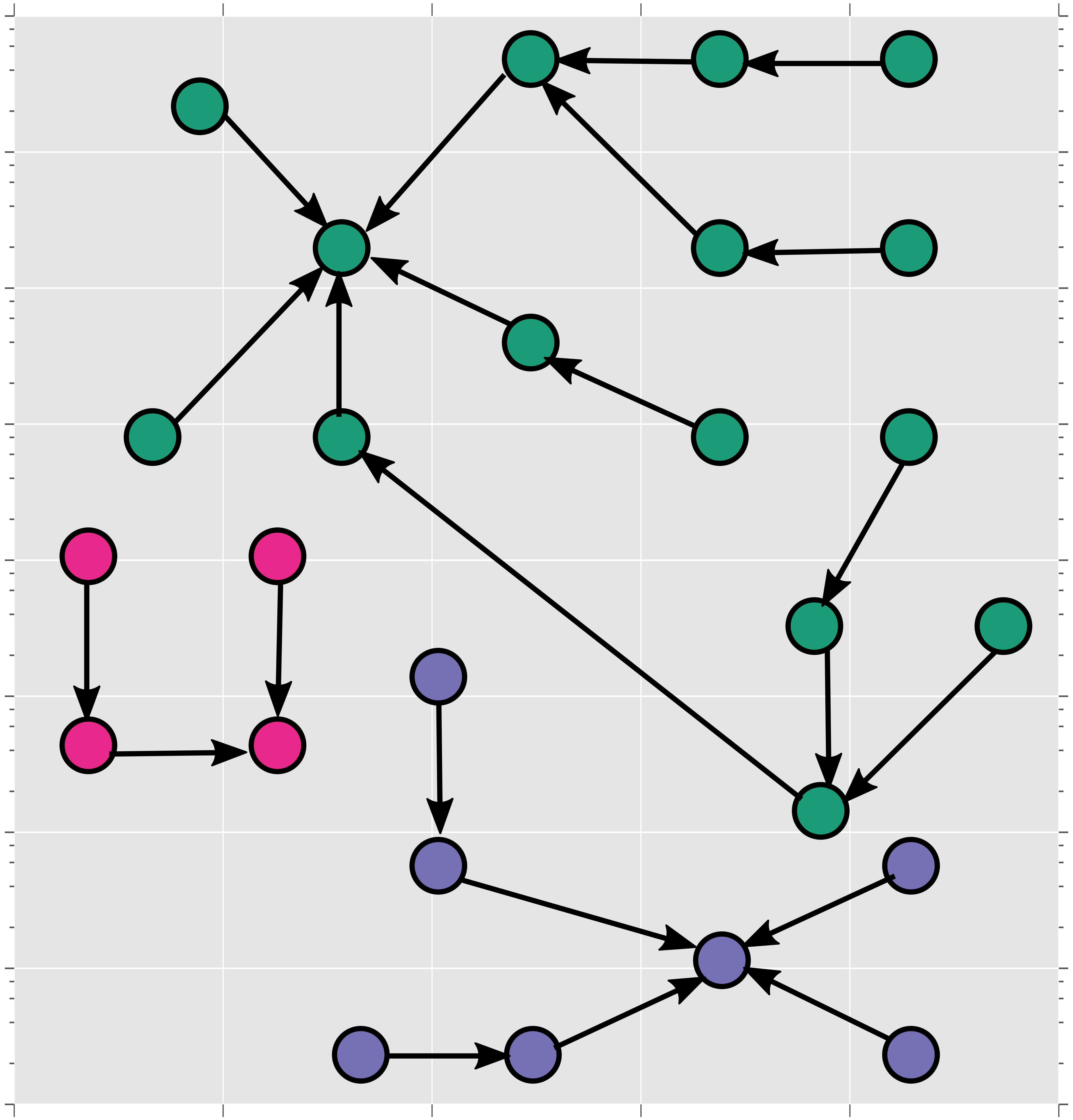
— Sao, Green, Jain, Vuduc (FTXS'16)

This Work

2. Self-stabilizing Connected-components

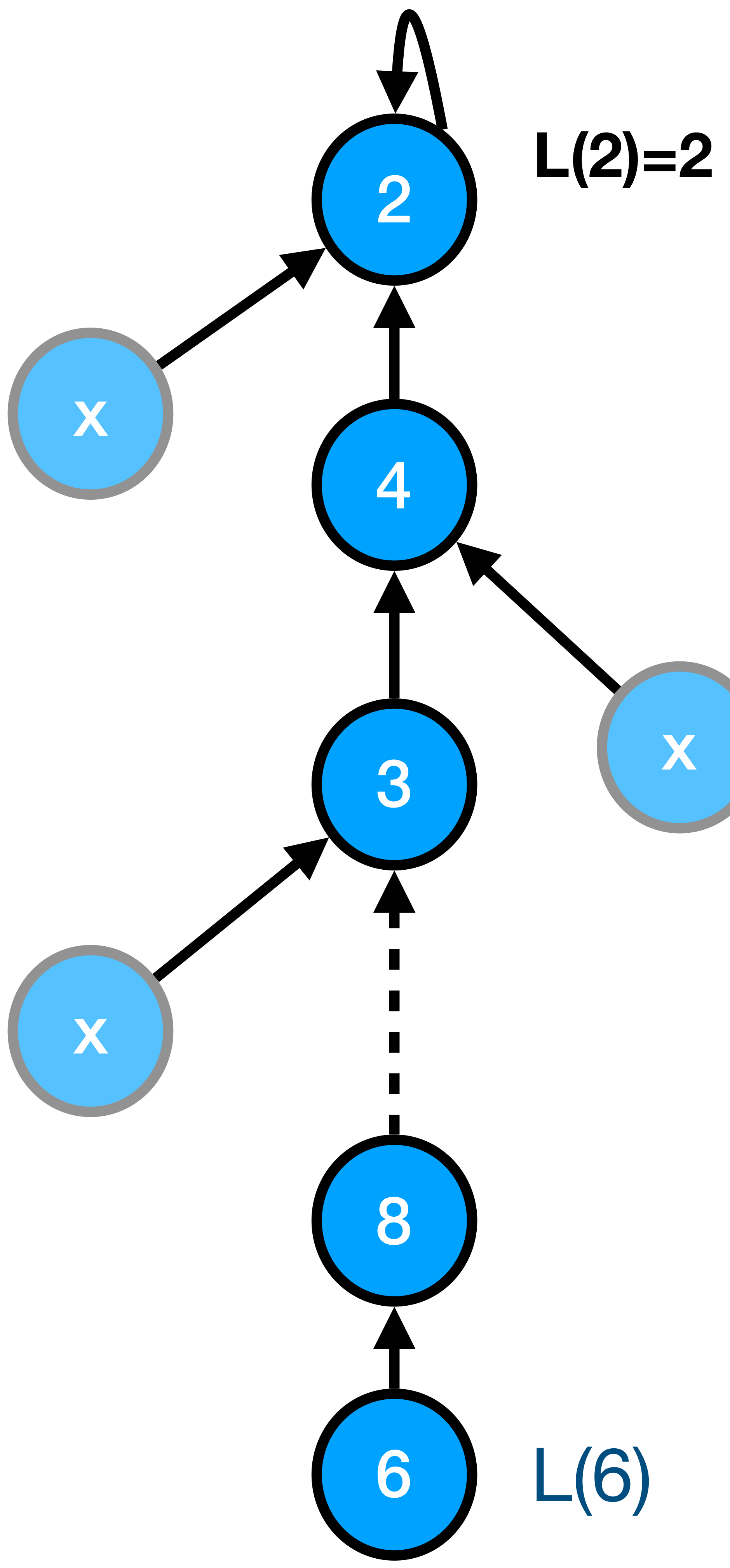
— Sao, Engalmann, Eswar, Green, Vuduc (FTXS'19)

Propagation Graph (H)



$$H = \{V, E_H\};$$
$$E_H = \{v \to P(v),\ \forall v \in V\}$$

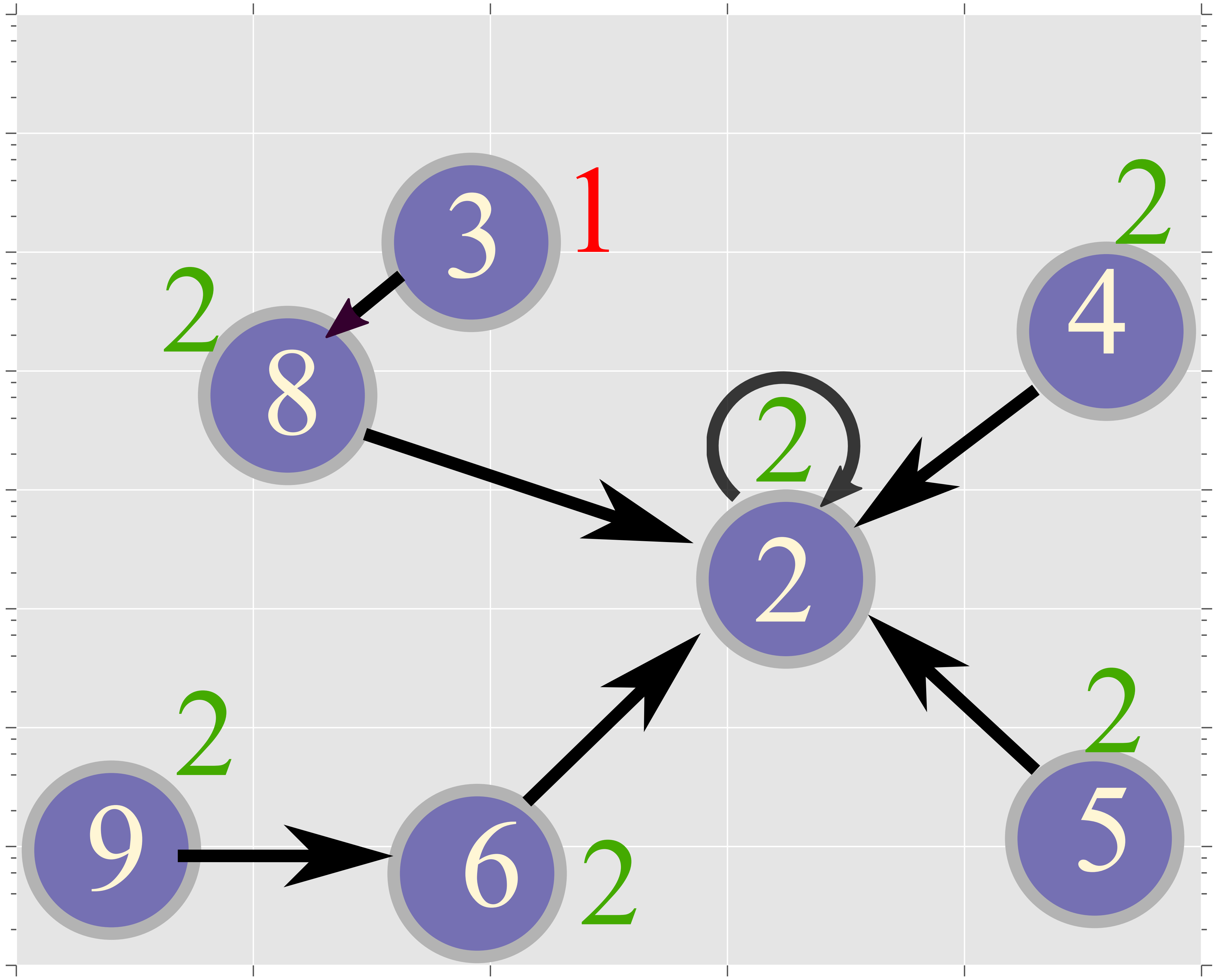
Properties of LP state



To verify:

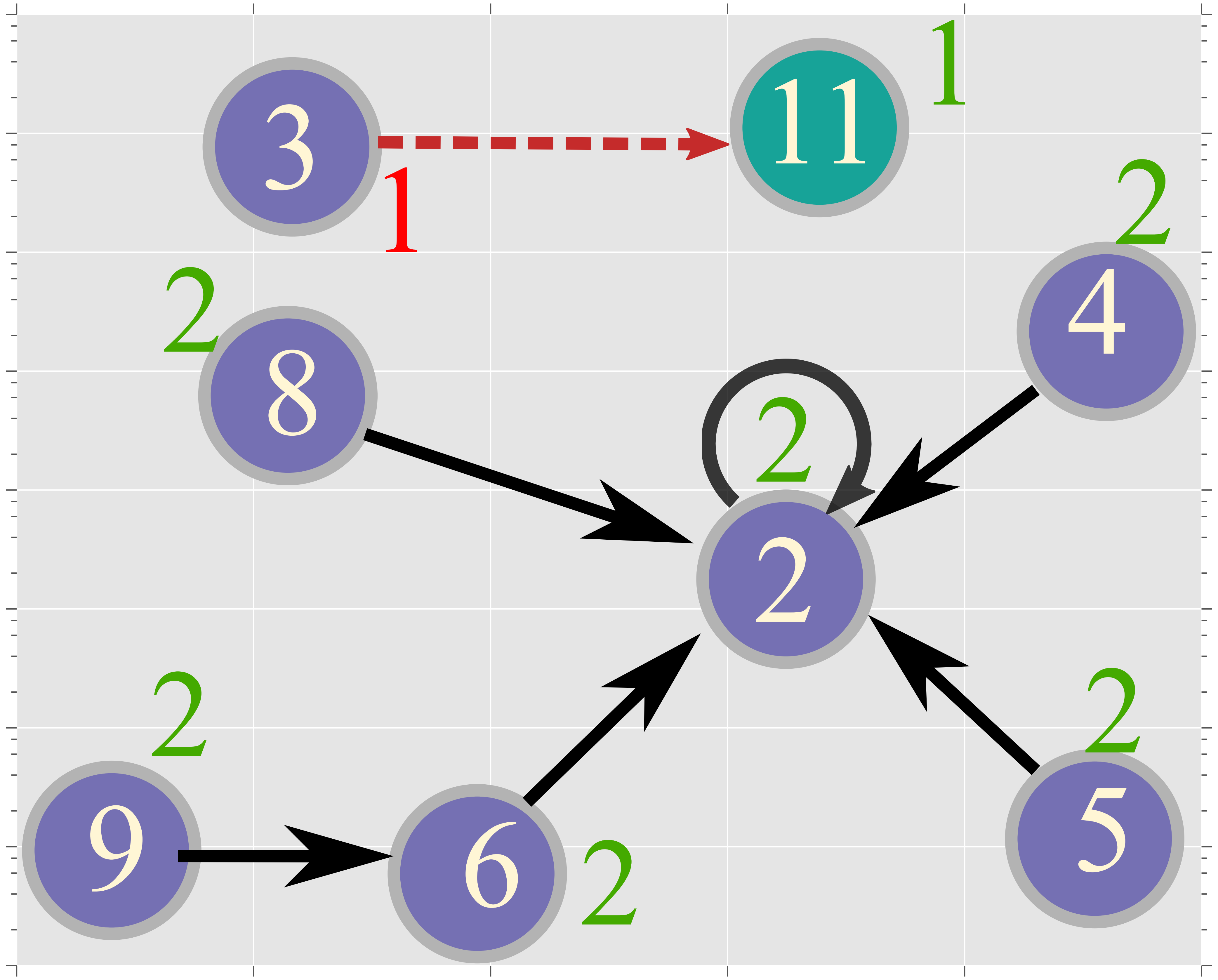
$$L^{\infty}(v) \leq L(v) \leq v \qquad \forall v \in V$$

Example-1



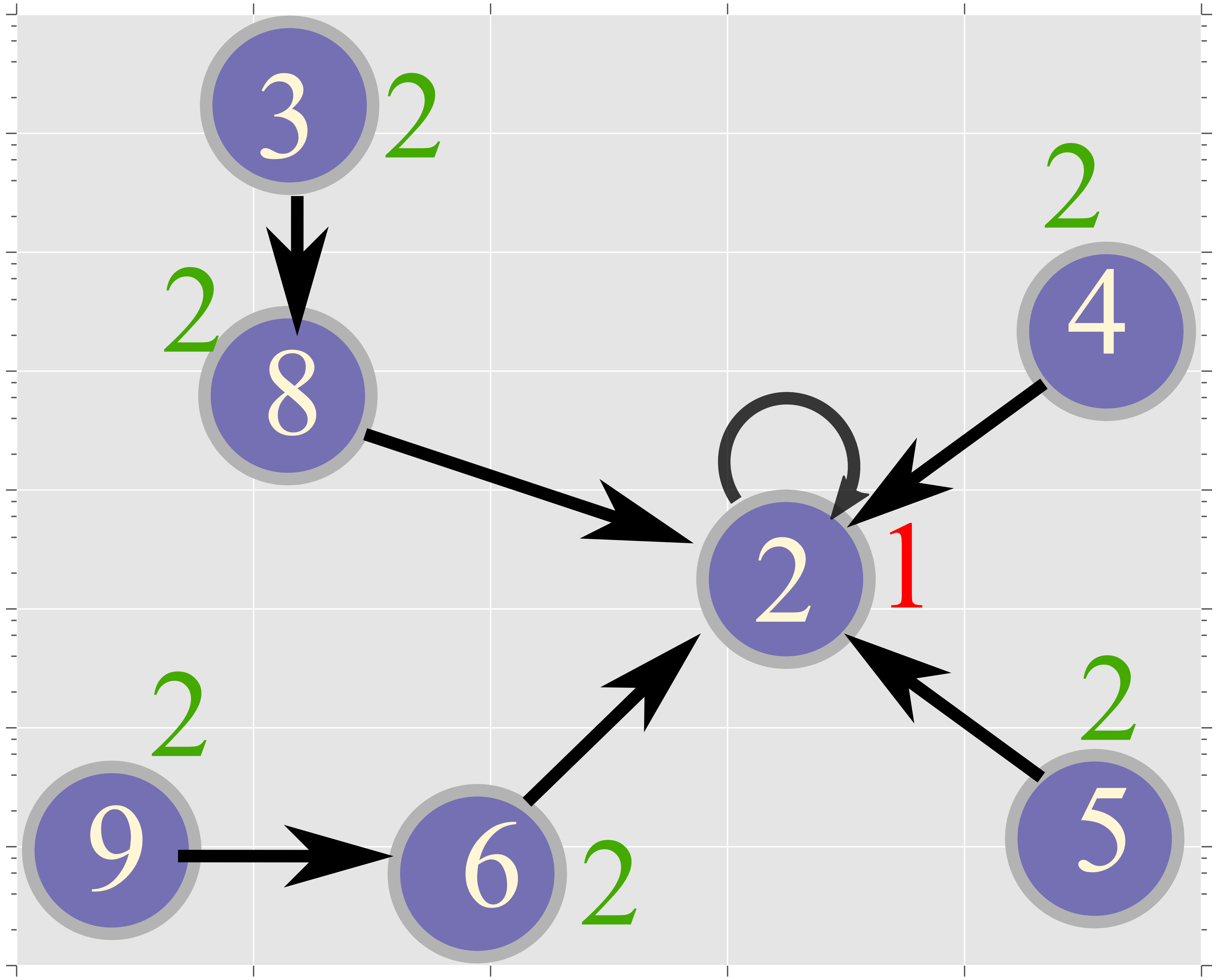
$$
\begin{array}{ccc}
L^i[v] & \leq & v; \\
P(v) & \in & \mathcal{N}(v) \\
\boxed{L^i[v] \quad \geq \quad L^i[P(v)]} & & \\
L^i[v] = v & \iff & P(v) = v
\end{array}
$$

Example-2



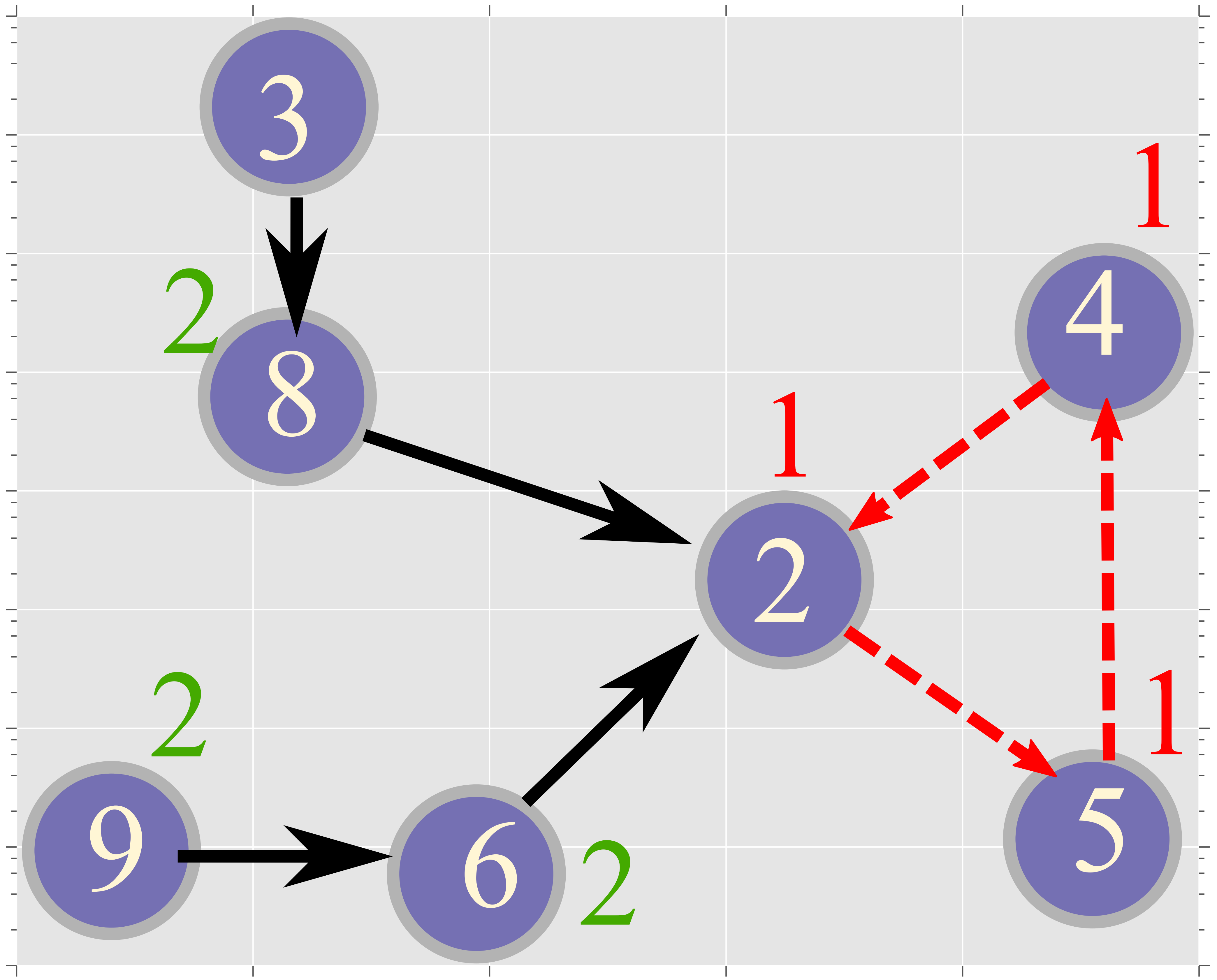
$$
\begin{aligned}
L^i[v] &\leq v; \\
\boxed{P(v)} &\boxed{\in \mathcal{N}(v)} \\
L^i[v] &\geq L^i[P(v)] \\
L^i[v] = v &\iff P(v) = v
\end{aligned}
$$

Example-3



$$L^i[v] \leq v;$$
$$P(v) \in \mathcal{N}(v)$$
$$L^i[v] \geq L^i[P(v)]$$
$$\boxed{L^i[v] = v \iff P(v) = v}$$

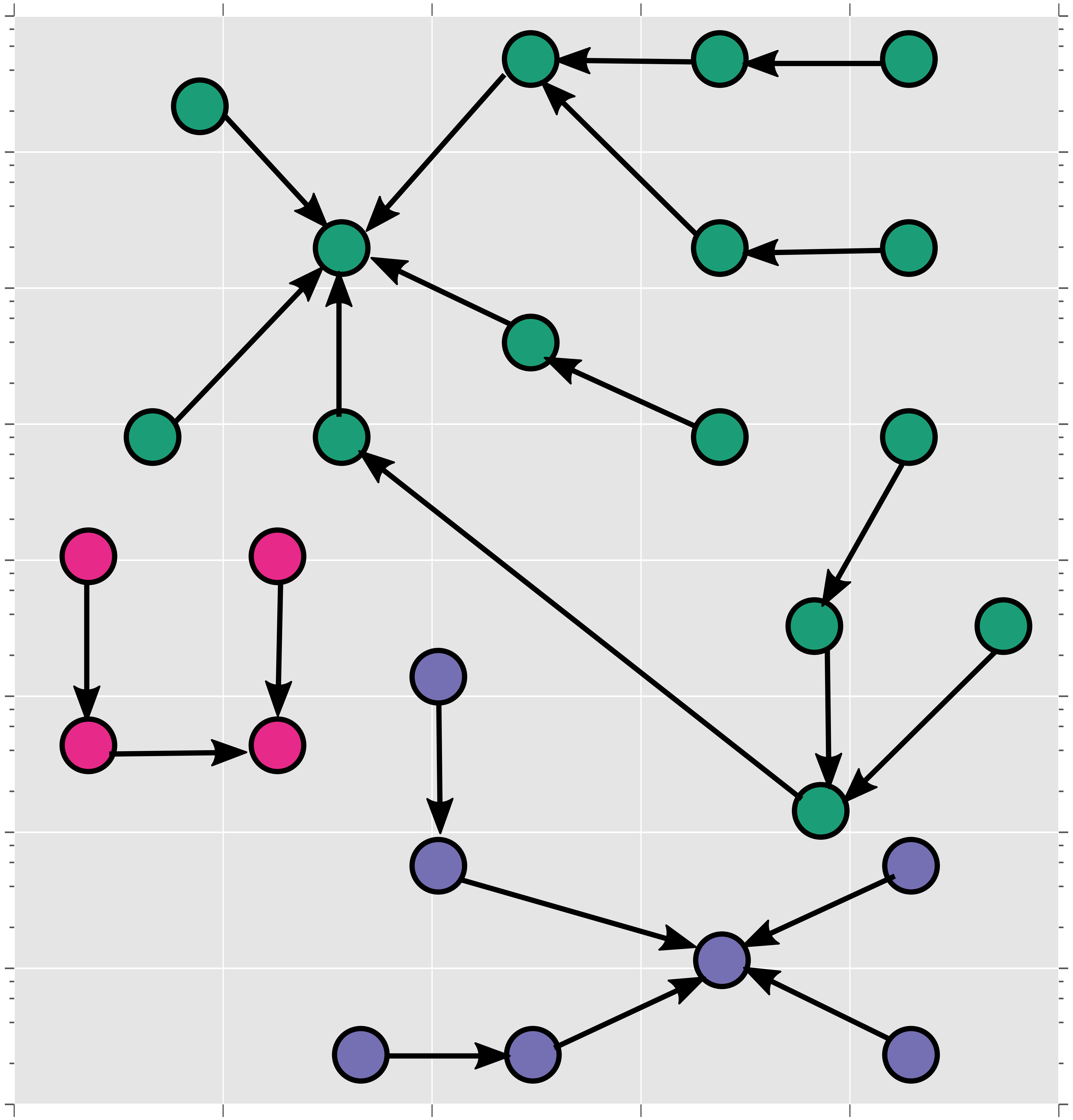
Example-4



$$
\begin{aligned}
L^i[v] &\leq v; \\
P(v) &\in \mathcal{N}(v) \\
L^i[v] &\geq L^i[P(v)] \\
L^i[v] = v &\iff P(v) = v
\end{aligned}
$$

Not covered by any condition ?

Valid States



S={L,P} is valid when:

$$\begin{aligned} L^i[v] &\leq v; \\ P(v) &\in \mathcal{N}(v) \\ L^i[v] &\geq L^i[P(v)] \\ L^i[v] = v &\Longleftrightarrow P(v) = v \\ \text{\#cycles}(H) &= \emptyset \end{aligned}$$

Detecting Cycles

S={L,P} is valid when:

$$\begin{array}{rcl} L^i[v] & \leq & v; \\ P(v) & \in & \mathcal{N}(v) \\ L^i[v] & \geq & L^i[P(v)] \\ L^i[v] = v & \Longleftrightarrow & P(v) = v \\ \text{\#cycles}(H) & = & \emptyset \end{array}$$

O(V log V)

$$\mathcal{A}(v) = \min \mathcal{P}(v) = \min \{P(v),\ P^2(v), \ldots\}$$

If $v = \mathcal{A}(v)$

Then

1) v is a vertex in a cycle in H; and
2) v has the smallest vertex-id in the cycle.

Self-stabilizing Connected Components

S={L,P} is valid when:

$$
\begin{aligned}
L^i[v] &\leq v; \\
P(v) &\in \mathcal{N}(v) \\
L^i[v] &\geq L^i[P(v)] \\
L^i[v] = v &\iff P(v) = v \\
\text{\#cycles}(H) &= \emptyset
\end{aligned}
$$

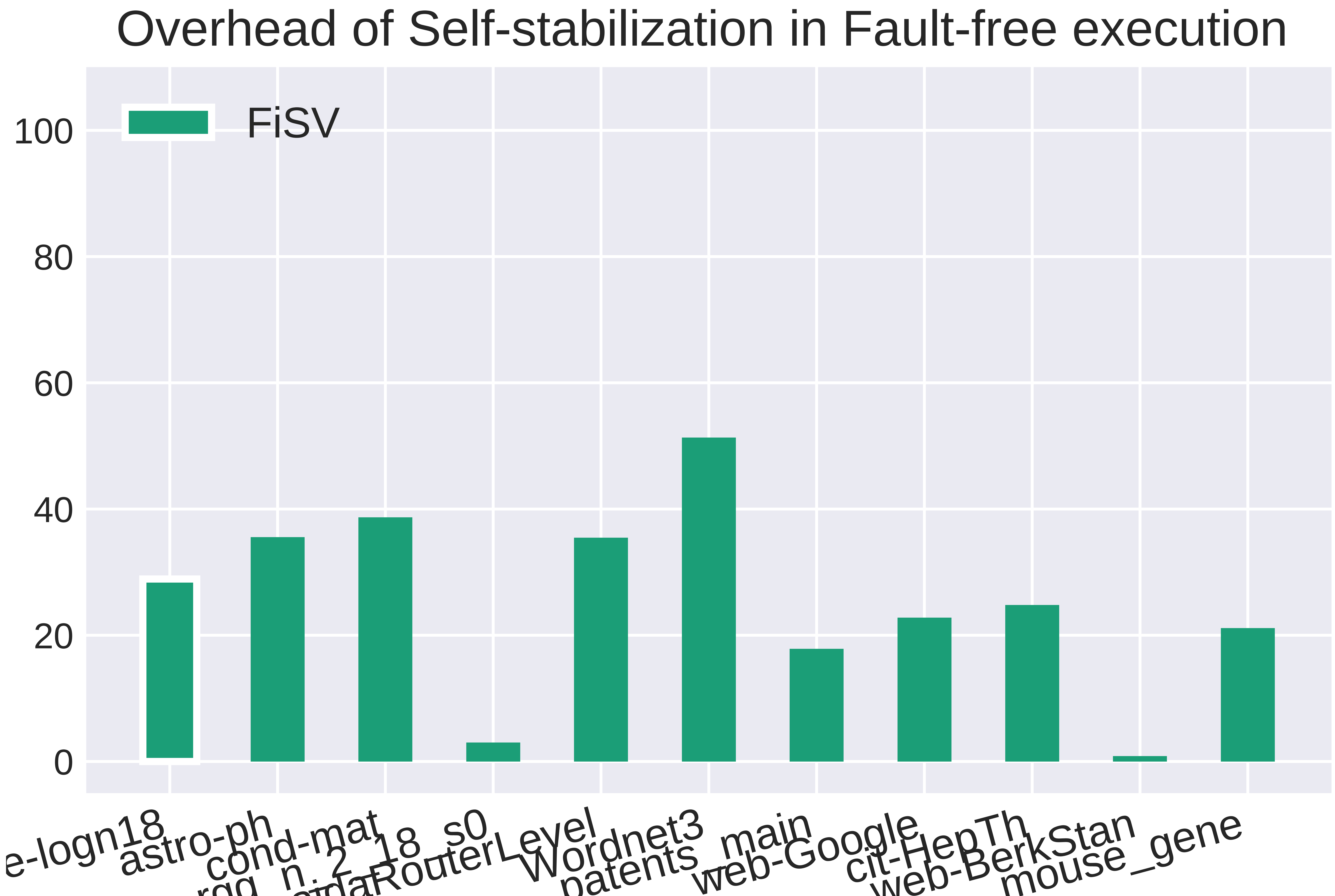
SsSV

Perform state check after the algorithm reports convergence

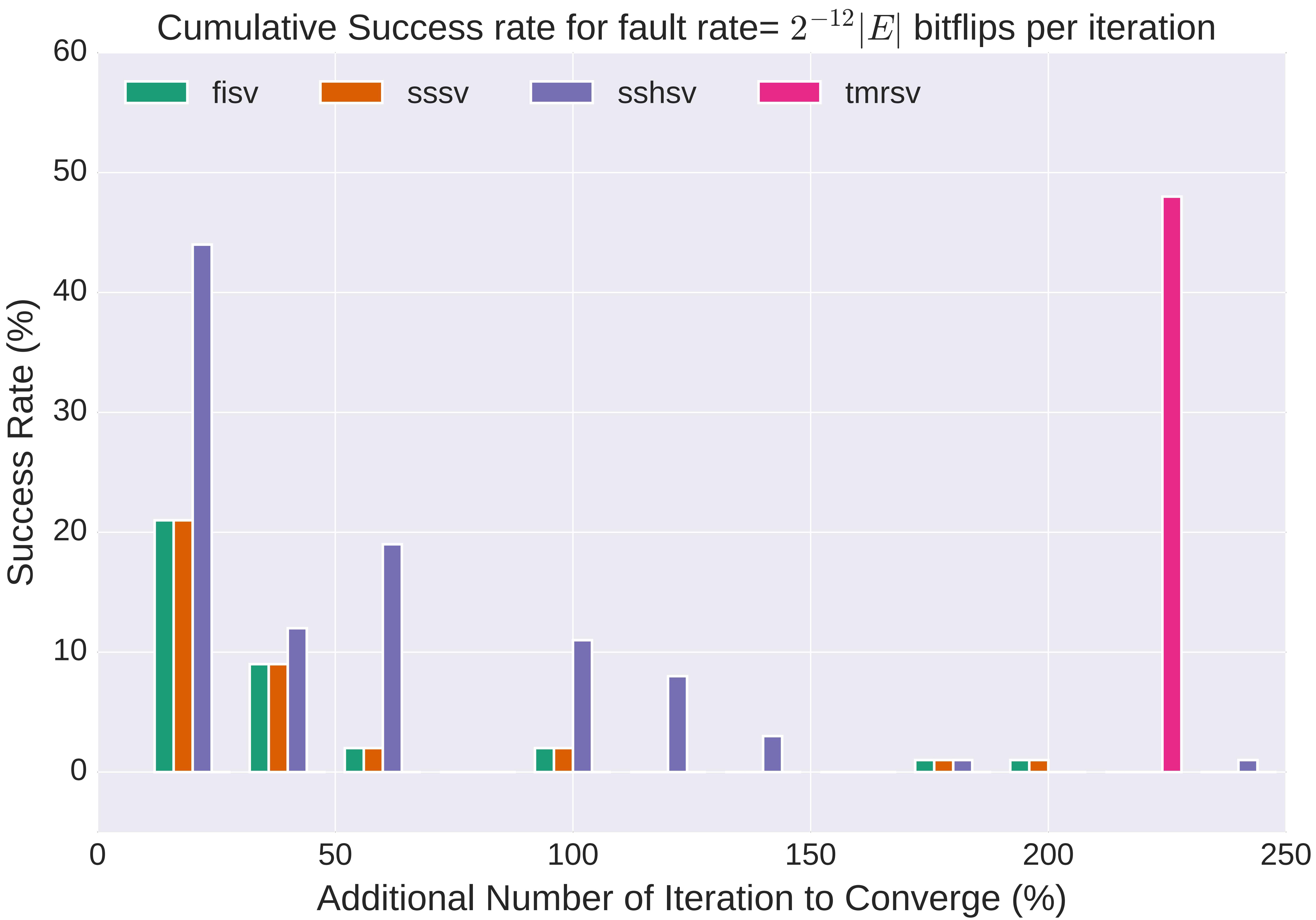
SsHSV

Perform local checks after every iteration and full state check after the algorithm reports convergence

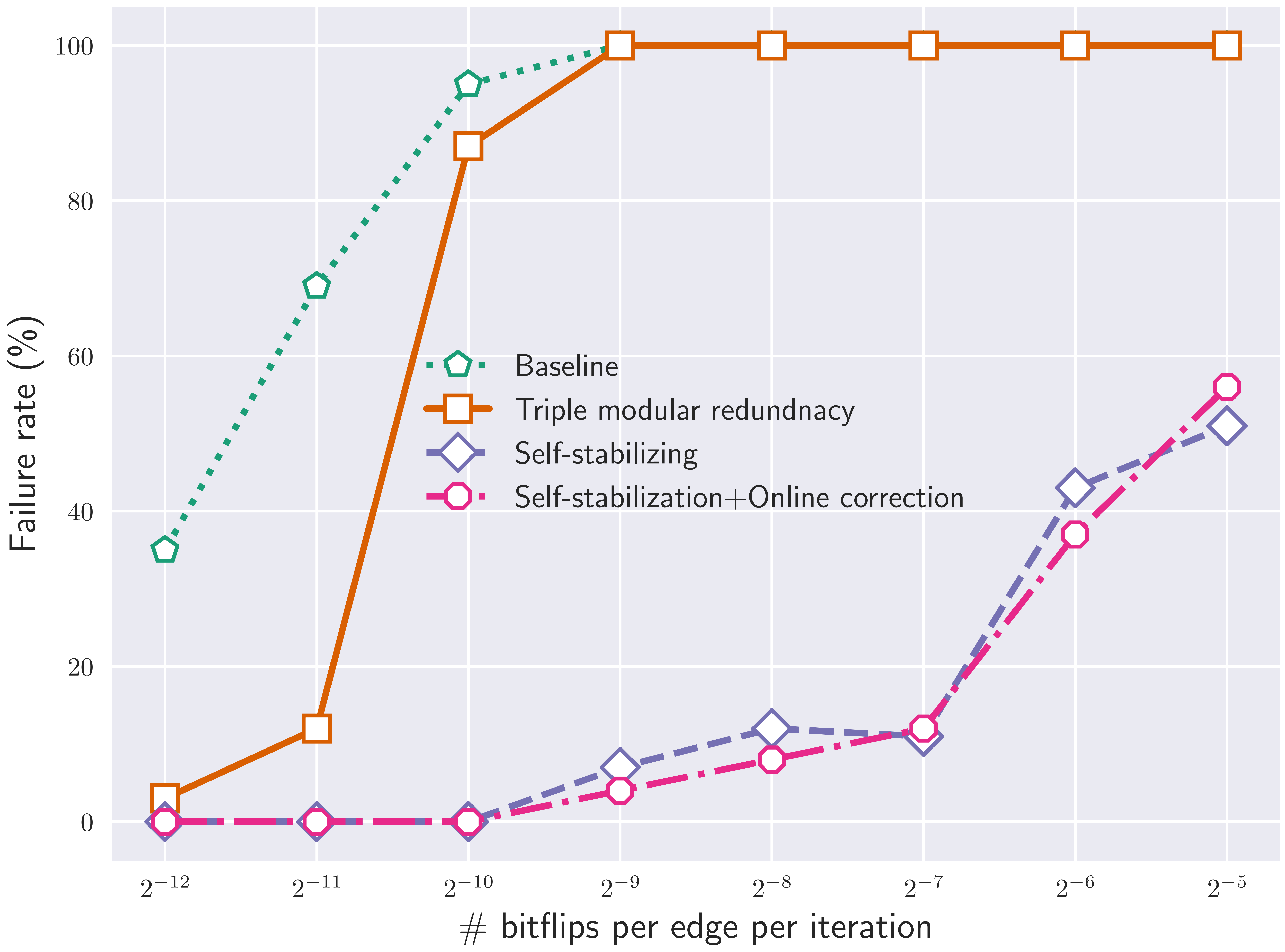
Overhead of Self-stabilization



Success Rate vs Additional Iterations



Failure Rate w.r.t. Fault Injection Rate



To sum up..

Conclusion

- Self-stabilization property of stationary iterations may not hold for graph algorithms (or semi-ring equivalent algorithms)
- Nevertheless, self-stabilization formulations may exists
- Efficiency of self-stabilization depends on the data structure

Future work

- Techniques used here are applicable to several other graph algorithms, e.g. BFS, Bellmen-Ford
- Self-stabilization could have practical use case in incremental/streaming graph processing